Question 11 (7 marks)

A factory advertises that its tea light candles burn for an average of minutes. The standard deviation of the burn times is known to be minutes.

(a) Quality control took a random sample of candles from the factory production line and recorded their burn times. These times were used to calculate the percent confidence interval for the population mean burn time as minutes. Determine the value of . (3 marks)

A consumer watchdog tested a random sample of candles made by the factory and their mean burn time was minutes.

(b) Describe and construct a suitable interval estimate based on this sample that can be used to advise the watchdog on the reasonableness of the factory’s advertising and use the interval estimate to provide that advice. (4 marks)

Question 11 (7 marks)

A factory advertises that its tea light candles burn for an average of minutes. The standard deviation of the burn times is known to be minutes.

(a) Quality control took a random sample of candles from the factory production line and recorded their burn times. These times were used to calculate the percent confidence interval for the population mean burn time as minutes. Determine the value of . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates standard error or margin of error  ü forms equation for -score  ü correct value of |

A consumer watchdog tested a random sample of candles made by the factory and their mean burn time was minutes.

(b) Describe and construct a suitable interval estimate based on this sample that can be used to advise the watchdog on the reasonableness of the factory’s advertising and use the interval estimate to provide that advice. (4 marks)

|  |
| --- |
| Solution |
| A suitable interval estimate is a confidence interval for the population mean burn time of a candle that can be constructed using the sample mean and known population standard deviation:  Interval estimate for population mean:  Advice to watchdog is that because the interval estimate for the mean burn time of all candles contains the advertised time of minutes, then the factory’s advertising is reasonable.  *Other intervals: and* . |
| Specific behaviours |
| ✓ correctly describes interval for population mean using confidence level  ü calculates variance of sampling distribution  ü constructs an interval estimate for population mean  ü advises watchdog, with reasoning, that advertised average is reasonable |

Question 16 (8 marks)

A machine fills bags with salt. The mean and standard deviation of the weight of salt it delivers into a bag is and grams respectively. An inspector routinely takes a random sample of bags filled by the machine.

(a) For repeated random sampling of bags of salt filled by this machine, state the approximate distribution of the sample mean that the inspector should expect. (3 marks)

(b) Determine the probability that the mean weight of a random sample of bags of salt is less than grams, given that the sample mean is greater than grams. (2 marks)

(c) Occasionally, the inspector only has enough time to take a random sample of bags. In the long run, of sample means derived from samples with this smaller size will lie in the range grams. Determine the value of . (3 marks)

Question 16 (8 marks)

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(a) For repeated random sampling of bags of salt filled by this machine, state the approximate distribution of the sample mean that the inspector should expect. (3 marks)

|  |
| --- |
| Solution |
| Let be the sample mean. Since the sample size is large then the distribution of will be approximately normal with mean g.  The standard deviation of is grams (variance)  Hence . |
| Specific behaviours |
| ✓ states that sample mean will be normally distributed  ü states the mean of the distribution  ü states the variance or standard deviation of the distribution |

(b) Determine the probability that the mean weight of a random sample of bags of salt is less than grams, given that the sample mean is greater than grams. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ forms correct probability statement  ü correct probability |

(c) Occasionally, the inspector only has enough time to take a random sample of bags. In the long run, of sample means derived from samples with this smaller size will lie in the range grams. Determine the value of . (3 marks)

|  |
| --- |
| Solution |
| The new standard deviation of is grams (variance). |
| Specific behaviours |
| ✓ states new parameters of distribution of sample mean  ü writes correct probability statement  ü correct value of |

Question 9 (8 marks)

The distribution of the weights of loaves of rye bread produced by a bakery has a mean and standard deviation of g and g respectively. Quality control frequently take random samples of white loaves from the bakery and calculate the mean weight of each sample.

(a) Describe the expected distribution of these sample means. (3 marks)

Further production checks are made if the mean weight of a sample is less than a prescribed minimum value of g.

(b) Over the course of the next random samples, how many times would you expect that further production checks need to be made? (2 marks)

Quality control has to reduce the sample size from to and change the prescribed minimum value so that the frequency of further production checks remains the same.

(c) Determine the prescribed minimum value for the mean weight of a sample required for this change. (3 marks)

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(a) Describe the expected distribution of these sample means. (3 marks)

|  |
| --- |
| Solution |
| Sample means will be normally distributed with a mean of g and standard deviation of g. |
| Specific behaviours |
| ✓ states normally distributed  ü states mean  ü calculations standard deviation |

Further production checks are made if the mean weight of a sample is less than a prescribed minimum value of g.

(b) Over the course of the next random samples, how many times would you expect that further production checks need to be made? (2 marks)

|  |
| --- |
| Solution |
| Hence, expect to make further checks times. |
| Specific behaviours |
| ✓ calculates probability  ü calculates expected number of times |

Quality control has to reduce the sample size from to and change the prescribed minimum value so that the frequency of further production checks remains the same.

(c) Determine the prescribed minimum value for the mean weight of a sample required for this change. (3 marks)

|  |
| --- |
| Solution |
| New standard deviation of sampling distribution will be .  Required -score for is . Hence  The prescribed minimum value should be changed to g. |
| Specific behaviours |
| ✓ calculations new standard deviation  ü obtains -score for required probability  ü calculates required value |

Question 13 (9 marks)

The mean and standard deviation of a random sample of physics teachers working in a region was and years respectively. The sample was taken to construct a confidence interval for the mean age of such teachers.

(a) State two reasons why it is appropriate to assume the approximate normality of the distribution of the sample mean for this data. (2 marks)

(b) State another assumption required to construct a valid confidence interval. (1 mark)

(c) Construct a confidence interval for the mean age of physics teachers working in the region. (3 marks)

(d) Based on another random sample, the confidence interval for the mean age of art teachers employed in the same region was calculated to be . Given that the standard deviation of the sample was years, determine the size of the sample.

(3 marks)

Question 13 (9 marks)

The mean and standard deviation of a random sample of physics teachers working in a region was and years respectively. The sample was taken to construct a confidence interval for the mean age of such teachers.

(a) State two reasons why it is appropriate to assume the approximate normality of the distribution of the sample mean for this data. (2 marks)

|  |
| --- |
| Solution |
| Sampling is random and sample size of is large (i.e., exceeds ). |
| Specific behaviours |
| ✓ states sampling is random  ü states sample size is large |

(b) State another assumption required to construct a valid confidence interval. (1 mark)

|  |
| --- |
| Solution |
| - sample standard deviation is a good estimate for the population standard deviation.  - sample values are independent of each other |
| Specific behaviours |
| ✓ states one valid assumption |

(c) Construct a confidence interval for the mean age of physics teachers working in the region. (3 marks)

|  |
| --- |
| Solution |
| Interval: |
| Specific behaviours |
| ✓ standard deviation of sampling distribution  ü correct expression for confidence interval  ü correct confidence interval |

(d) Based on another random sample, the confidence interval for the mean age of art teachers employed in the same region was calculated to be . Given that the standard deviation of the sample was years, determine the size of the sample.

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ margin of error  ü correct equation for sample size  ü correct sample size |

Question 11 (7 marks)

In order to estimate the mean cost of damage sustained by parked vehicles when struck by another vehicle, an insurance company examined the records of such occurrences, and obtained a sample mean of with sample standard deviation .

(a) Construct a confidence interval for the mean cost of damage in all such accidents.

(3 marks)

(b) Previously, the insurance company had used the amount of for the mean cost of damage in all such accidents. State, with reasons, whether this amount is no longer valid.

(2 marks)

(c) State one assumption made in constructing the interval in part (a) and comment on how reasonable this assumption is in relation to the information provided. (2 marks)

Question 11 (7 marks)

In order to estimate the mean cost of damage sustained by parked vehicles when struck by another vehicle, an insurance company examined the records of such occurrences, and obtained a sample mean of with sample standard deviation .

(a) Construct a confidence interval for the mean cost of damage in all such accidents.

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates standard deviation for sample mean  ü indicates calculations for bounds of interval  ü correct interval, to nearest dollar |

(b) Previously, the insurance company had used the amount of for the mean cost of damage in all such accidents. State, with reasons, whether this amount is no longer valid.

(2 marks)

|  |
| --- |
| Solution |
| There is no reason to doubt the validity of this amount as it is contained within the bounds of the confidence interval. |
| Specific behaviours |
| ✓ states no reason to doubt validity  ü states amount contained within interval |

(c) State one assumption made in constructing the interval in part (a) and comment on how reasonable this assumption is in relation to the information provided. (2 marks)

|  |  |  |
| --- | --- | --- |
| Solution | | |
| Sample means are normally distributed.  Reasonable due to largish sample size of . | Sample was obtained randomly.  Cannot comment as no information provided on how sample collected. | Sample values are independent of each other.  Reasonable as such accidents are unlikely to be related. |
| Specific behaviours | | |
| ✓ states assumption  ü comments on reasonableness | | |

Question 15 (7 marks)

The mass of raisins in each g packet of muesli produced by a company is normally distributed with a mean of g and standard deviation g.

(a) Determine the probability that the total mass of raisins in a random sample of packets of muesli is at least g. (4 marks)

(b) Another random sample of packets is to be taken. Determine the minimum sample size required so that the chance that the sample mean mass of raisins is less than g will be no more than . (3 marks)

Question 15 (7 marks)

The mass of raisins in each g packet of muesli produced by a company is normally distributed with a mean of g and standard deviation g.

(a) Determine the probability that the total mass of raisins in a random sample of packets of muesli is at least g. (4 marks)

|  |
| --- |
| Solution |
| Let be the sample mean of packets.  is normally distributed with mean g and standard deviation g. |
| Specific behaviours |
| ✓ states sample means normally distributed  ü states parameters of normal distribution  ü indicates sample mean required  ü calculates probability |

(b) Another random sample of packets is to be taken. Determine the minimum sample size required so that the chance that the sample mean mass of raisins is less than g will be no more than . (3 marks)

|  |
| --- |
| Solution |
| is now distributed with mean g and standard deviation .  Require:  Hence minimum sample size of is required. |
| Specific behaviours |
| ✓ probability statement  ü forms inequality for using correct -score  ü states minimum integer value |

Question 9 (6 marks)

The time interval between vehicles arriving at a -hour service station is known to follow an exponential distribution with a standard deviation of seconds.

The mean of a random sample of time intervals was seconds.

(a) Use the sample to construct a confidence interval for the mean of . (4 marks)

(b) State the key assumption made when constructing the interval in part (a) and explain how confident you are that the assumption is valid. (2 marks)

Question 9 (6 marks)

The time interval between vehicles arriving at a -hour service station is known to follow an exponential distribution with a standard deviation of seconds.

The mean of a random sample of time intervals was seconds.

(a) Use the sample to construct a confidence interval for the mean of . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sample mean as interval centre   calculates standard error   uses correct -score for confidence level   correct bounds of interval |

(b) State the key assumption made when constructing the interval in part (a) and explain how confident you are that the assumption is valid. (2 marks)

|  |
| --- |
| **Solution** |
| Assumed that the distribution of sample means is normal.  Although the population distribution is not normal, because the sample size is large (greater than ) we can be confident that the distribution of sample means will approximate normality. |
| **Specific behaviours** |
| ✓ states correct assumption   states confident and refers to large sample size |

Question 14 (10 marks)

The shell weight of eggs laid by hens in a flock is known to be normally distributed with mean of g and standard deviation g.

(a) A random sample of eggs is selected from the flock and the mean shell weight of these eggs calculated.

(i) State the distribution of , the sample mean. (3 marks)

(ii) Determine the probability that the sample mean is between g and g.

(2 marks)

(iii) Suppose the size of the random sample was halved. Explain, without any further calculation, how this will affect your answer to part (ii). (2 marks)

(b) Random samples of eggs were repeatedly selected from the flock and the mean weight of each sample recorded. It was observed that of the sample means weighed less than g. Determine the value of .

Question 14 (10 marks)

The shell weight of eggs laid by hens in a flock is known to be normally distributed with mean of g and standard deviation g.

(a) A random sample of eggs is selected from the flock and the mean shell weight of these eggs calculated.

(i) State the distribution of , the sample mean. (3 marks)

|  |
| --- |
| **Solution** |
| Sample means will be normally distributed as population is normally distributed.  Mean of is g and variance is g2 (sd g). |
| **Specific behaviours** |
| ✓ states sample mean is normally distributed   states correct mean of distribution   states correct variance (or sd) of distribution |

(ii) Determine the probability that the sample mean is between g and g.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct -scores   correct probability |

(iii) Suppose the size of the random sample was halved. Explain, without any further calculation, how this will affect your answer to part (ii). (2 marks)

|  |
| --- |
| **Solution** |
| The probability will decrease.  With half the sample size, the variance of the sampling distribution will double and so the range of -scores for the interval will be smaller, leading to a lower probability. |
| **Specific behaviours** |
| ✓ states probability will decrease   justifies with increased spread or smaller range of -scores |

(b) Random samples of eggs were repeatedly selected from the flock and the mean weight of each sample recorded. It was observed that of the sample means weighed less than g. Determine the value of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates -score for required probability   indicates calculation for sample sd   correct value of as integer |

Question 12 (8 marks)

The diameter of copper wire produced by a machine is normally distributed with a mean of and a variance of .

A production supervisor routinely takes a random sample of diameters and calculates their mean, .

(a) Describe the distribution of . (3 marks)

(b) Determine the probability that the mean of a random sample of diameters is less than . (1 mark)

(c) Repeated random sampling of diameters from the machine shows that there is a chance that the sample mean exceeds . Determine . (4 marks)

Question 12 (8 marks)

The diameter of copper wire produced by a machine is normally distributed with a mean of and a variance of .

A production supervisor routinely takes a random sample of diameters and calculates their mean, .

(a) Describe the distribution of . (3 marks)

|  |
| --- |
| **Solution** |
| is normally distributed |
| **Specific behaviours** |
| ✓ states normally distributed   states mean   states variance (or sd) |

(b) Determine the probability that the mean of a random sample of diameters is less than . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(c) Repeated random sampling of diameters from the machine shows that there is a chance that the sample mean exceeds . Determine . (4 marks)

|  |
| --- |
| **Solution 2** |
|  |
| **Specific behaviours** |
| ✓ writes distribution   indicates use of -score,   writes equation for   correct value of |

|  |
| --- |
| **Solution 1** |
|  |
| **Specific behaviours** |
| ✓ writes distribution  ✓ writes probability statement   writes equation for   correct value of |

Question 15 (8 marks)

A researcher used data from a sample of newborn babies in order to estimate the mean weight and length of newborns in a large city.

(a) The weights of the babies in the sample had a mean of kg and a standard deviation of kg.

(i) Use this data to obtain a confidence interval for the mean weight of a newborn baby in the city. (2 marks)

(ii) State two assumptions made when constructing your confidence interval. (2 marks)

(b) The confidence interval for the mean length cm of newborn babies derived from the sample was . Determine the sample mean and standard deviation used to construct this interval. (4 marks)

Question 15 (8 marks)

A researcher used data from a sample of newborn babies in order to estimate the mean weight and length of newborns in a large city.

(a) The weights of the babies in the sample had a mean of kg and a standard deviation of kg.

(i) Use this data to obtain a confidence interval for the mean weight of a newborn baby in the city. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ lower bound   upper bound |

(ii) State two assumptions made when constructing your confidence interval. (2 marks)

|  |
| --- |
| **Solution** |
| Sample was obtained randomly  Sample values are independent of each other  Sample means are normally distributed |
| **Specific behaviours** |
| ✓ first assumption   second assumption |

(b) The confidence interval for the mean length cm of newborn babies derived from the sample was . Determine the sample mean and standard deviation used to construct this interval. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ margin of error   mean   equation for sd   standard deviation |

Question 12 (8 marks)

(a) Bags of lemons are packaged for sale by a supermarket. The population mean and standard deviation of the weight of the bags is known to be kg and g respectively.

Determine the probability that the total weight of a random sample of bags of lemons is greater than kg. (4 marks)

(b) The supermarket also packs bags of oranges for sale. The weights of the bags have a population mean and standard deviation of and kg respectively.

A random sample of bags was taken and used to construct a confidence interval for . If the interval was , determine an estimate for . (4 marks)

Question 12 (8 marks)

(a) Bags of lemons are packaged for sale by a supermarket. The population mean and standard deviation of the weight of the bags is known to be kg and g respectively.

Determine the probability that the total weight of a random sample of bags of lemons is greater than kg. (4 marks)

|  |
| --- |
| **Solution** |
| Let be the distribution of random samples of size 3 from the population.  Then |
| **Specific behaviours** |
| ✓ defines sample mean as a normally distributed rv   indicates parameters of normal distribution   indicates probability calculated   correct probability |

(b) The supermarket also packs bags of oranges for sale. The weights of the bags have a population mean and standard deviation of and kg respectively.

A random sample of bags was taken and used to construct a confidence interval for . If the interval was , determine an estimate for . (4 marks)

|  |
| --- |
| **Solution** |
| Margin of error: |
| **Specific behaviours** |
| ✓ calculates margin of error   uses correct -score   writes equation for margin of error   correct standard deviation |

Question 16 (9 marks)

The durations, in minutes, of a sample of calls to an IT support line were as follows.

The duration of calls to the support line has a known standard deviation of minutes seconds.

(a) Stating two necessary assumptions, construct a confidence interval for the mean duration of calls to the support line. (7 marks)

(b) Comment, with justification, on a claim that the mean duration of calls to the support line is minutes. (2 marks)

Question 16 (9 marks)

The durations, in minutes, of a sample of calls to an IT support line were as follows.

The duration of calls to the support line has a known standard deviation of minutes seconds.

(a) Stating two necessary assumptions, construct a confidence interval for the mean duration of calls to the support line. (7 marks)

|  |
| --- |
| **Solution** |
| (i) **Sample** is **random**  (ii) **Durations** are **normal**(ly distributed) |
| **Specific behaviours** |
| ✓ assumption (i) (must use both bolded words)   assumption (ii) (must use both bolded words)   calculates sample mean   indicates correct -score   indicates correct standard error   indicates interval construct   calculates interval within ranges shown |

(b) Comment, with justification, on a claim that the mean duration of calls to the support line is minutes. (2 marks)

|  |
| --- |
| **Solution** |
| minutes lies within the CI and so claim is reasonable. |
| **Specific behaviours** |
| ✓ refers to relative to CI   comment supported by reference |

Question 11 (6 marks)

When used in a torch, the lifetime of a single AAA battery was observed to be normally distributed with a mean of hours and a standard deviation of hours.

A student bought 40 boxes of these batteries, with 48 batteries in each box, and calculated the average lifetime for the batteries in each box. The mean of the averages was 8.31 hours and the standard deviation of the averages was 0.05 hours.

(a) Use this information to determine estimates for and . (3 marks)

(b) The batteries in one of the boxes lasted for a total of 396 hours. Use this sample of 48 batteries to construct a 95% confidence interval for the lifetime of this type of AAA battery. (3 marks)

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(a) Use this information to determine estimates for and . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sample mean as best estimate for  ✓ indicates sample sd smaller by factor  ✓ correct estimate for |

(b) The batteries in one of the boxes lasted for a total of 396 hours. Use this sample of 48 batteries to construct a 95% confidence interval for the lifetime of this type of AAA battery. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates mean  ✓ calculates margin of error  ✓ states interval |

Question 17 (9 marks)

The serving sizes of coffee dispensed by a machine have been observed to have a mean of 140 mL and a standard deviation of 2.8 mL.

(a) A random sample of 60 serves of coffee are taken from the machine and the serving size is measured in each case. Determine the probability that

(i) the sample mean will be at least 140.5 mL. (3 marks)

(ii) the total amount of coffee dispensed will be between 8.34 L and 8.46 L. (3 marks)

(b) After servicing of the machine, an inspector plans to construct a 98% confidence interval for the serving size dispensed by the machine. Determine the sample size they should take so that the width of the interval is no more than 1 mL, and note any assumptions made. (3 marks)

Question 17 (9 marks)

The serving sizes of coffee dispensed by a machine have been observed to have a mean of 140 mL and a standard deviation of 2.8 mL.

(a) A random sample of 60 serves of coffee are taken from the machine and the serving size is measured in each case. Determine the probability that

(i) the sample mean will be at least 140.5 mL. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates sample mean is normal rv  ✓ states correct parameters of normal distribution  ✓ states probability |

(ii) the total amount of coffee dispensed will be between 8.34 L and 8.46 L. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates sample mean serving sizes  ✓ writes statement of probability calculated  ✓ states probability |

(b) After servicing of the machine, an inspector plans to construct a 98% confidence interval for the serving size dispensed by the machine. Determine the sample size they should take so that the width of the interval is no more than 1 mL, and note any assumptions made. (3 marks)

|  |
| --- |
| **Solution** |
| Assumed: standard deviation is still 2.8 mL, approximate normality of sampling distribution. |
| **Specific behaviours** |
| ✓ indicates correct z-score and interval half-width  ✓ calculates sample size as integer  ✓ notes at least one valid assumption |